

# Enforcing optimal operation of FCS despite degradation via real-time optimization

Tafarel de Avila Ferreira

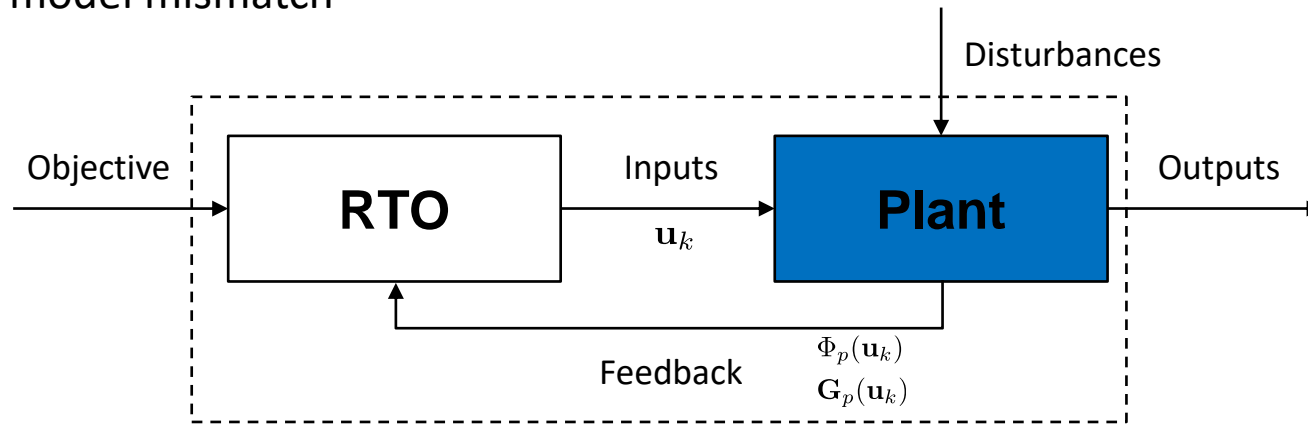
Workshop – From basic to applied research towards durable and reliable fuel cells

July 5th, 2022  
Lucerne, Switzerland

# Real-Time Optimization

Techniques that use **process measurements** to improve plant performance in the presence of

- Disturbances
- Plant-model mismatch



## Static Real-Time Optimization

- Adaptation of cost and constraint functions – **Modifier adaptation**

# The Role of Model in RTO

## Plant

$$\begin{aligned} \min_{\mathbf{u}} \quad & \Phi_p(\mathbf{u}) := \phi(\mathbf{u}, \mathbf{y}_p(\mathbf{u})) \\ \text{s.t.} \quad & G_{p,i}(\mathbf{u}) := g_i(\mathbf{u}, \mathbf{y}_p(\mathbf{u})) \leq 0 \\ & i = 1, \dots, n_g \\ & \mathbf{u} \in \mathcal{U} \end{aligned}$$

## Model

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## Challenge

- **Optimal plant** operation!
- **Uncertain** model, i.e.  $\phi_p(u) \neq \phi(u, \theta)$ ,  $G_{p,i}(u) \neq G_i(u, \theta)$ .

## Real-Time Optimization

- Use process measurements
- Which entities should be measured?
- How should these measurements be used?

# The Role of Model in RTO

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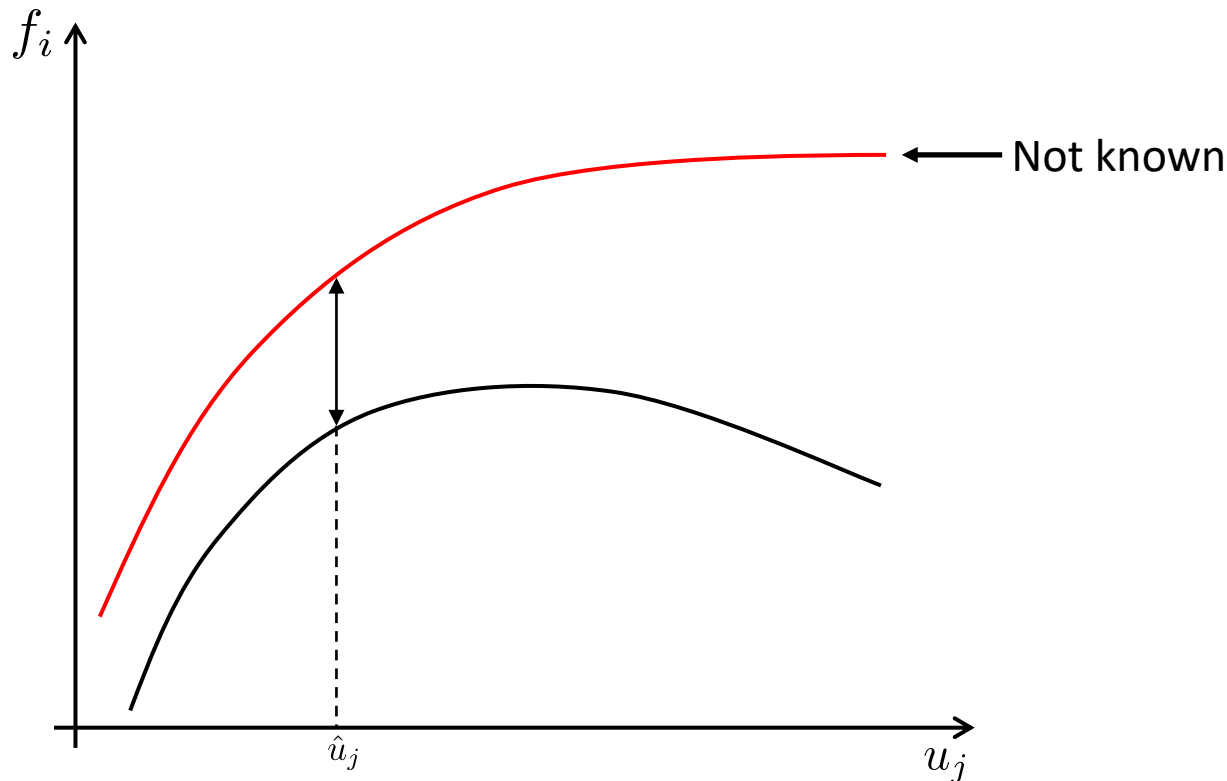
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# Basic Features of Modifier Adaptation

How is the optimization scheme adapted?

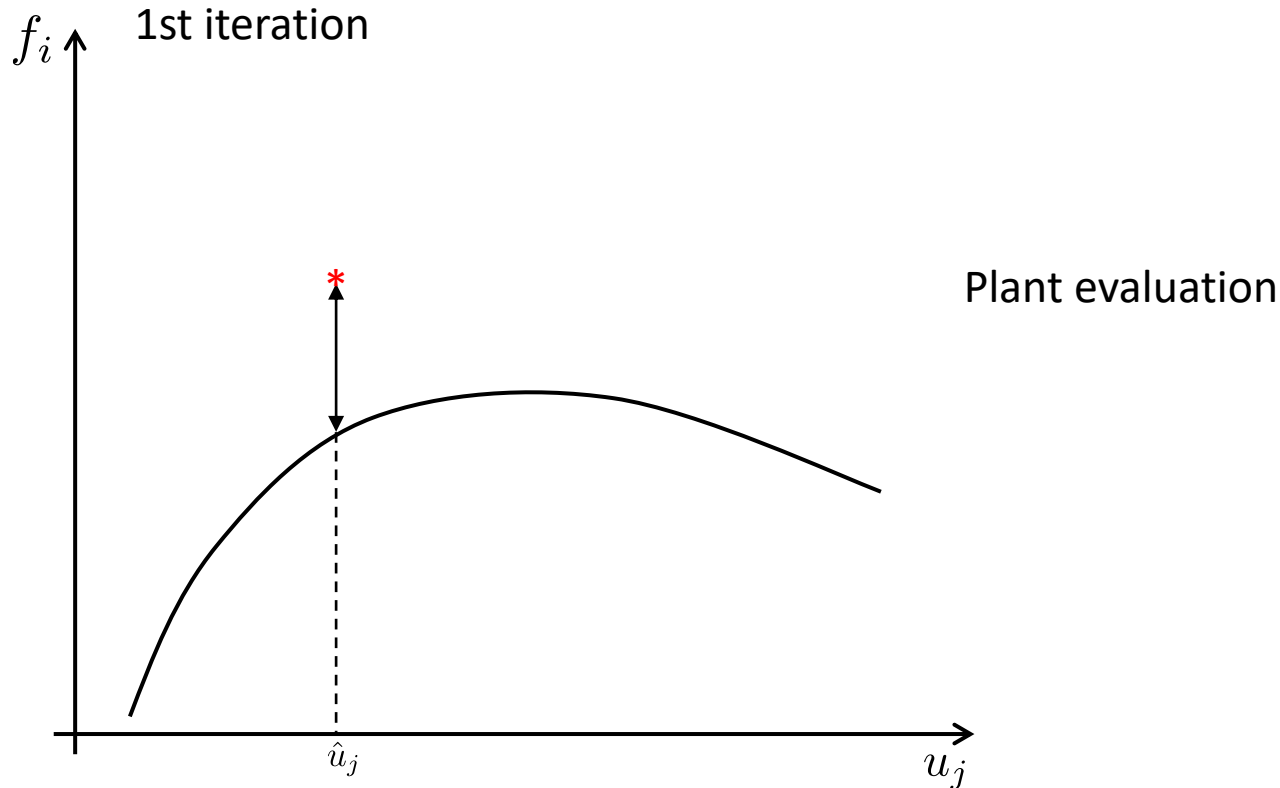
Modifier Adaptation: Zeroth- and first-order correction terms added to cost and constraint functions to the optimization problem



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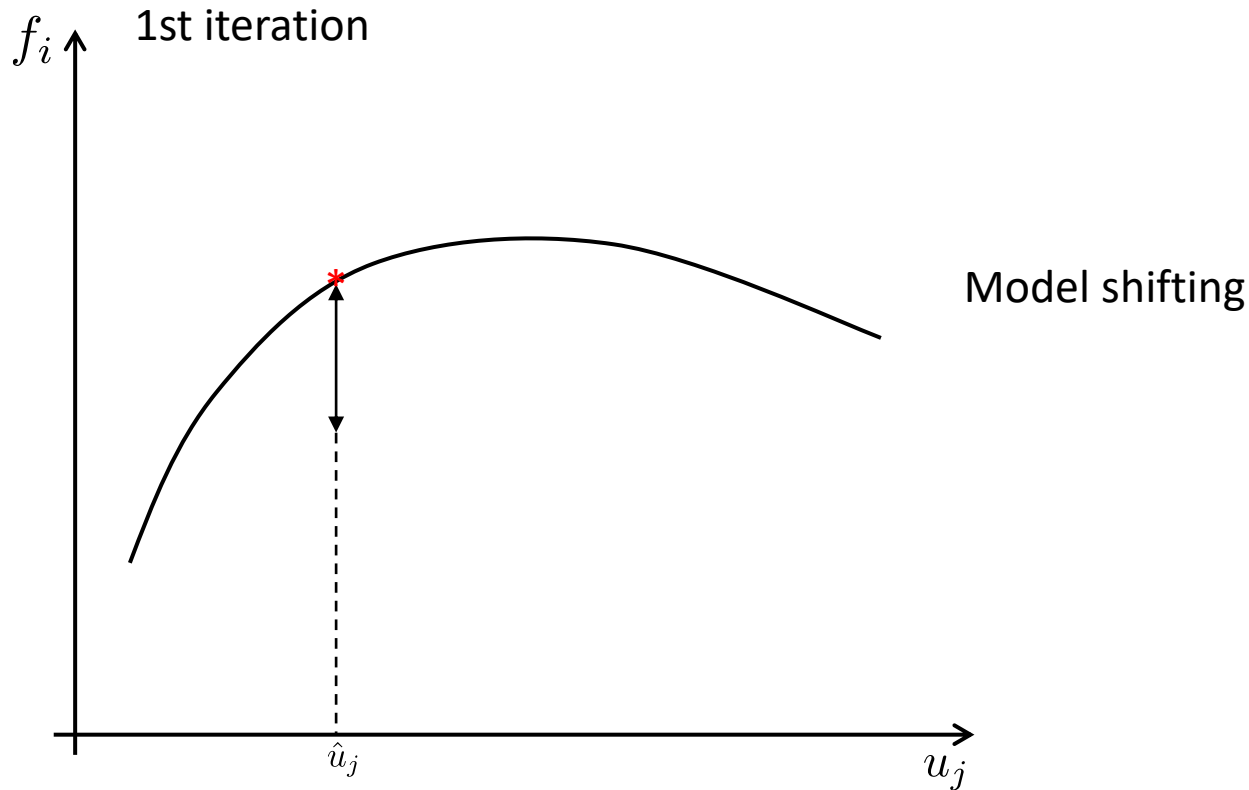
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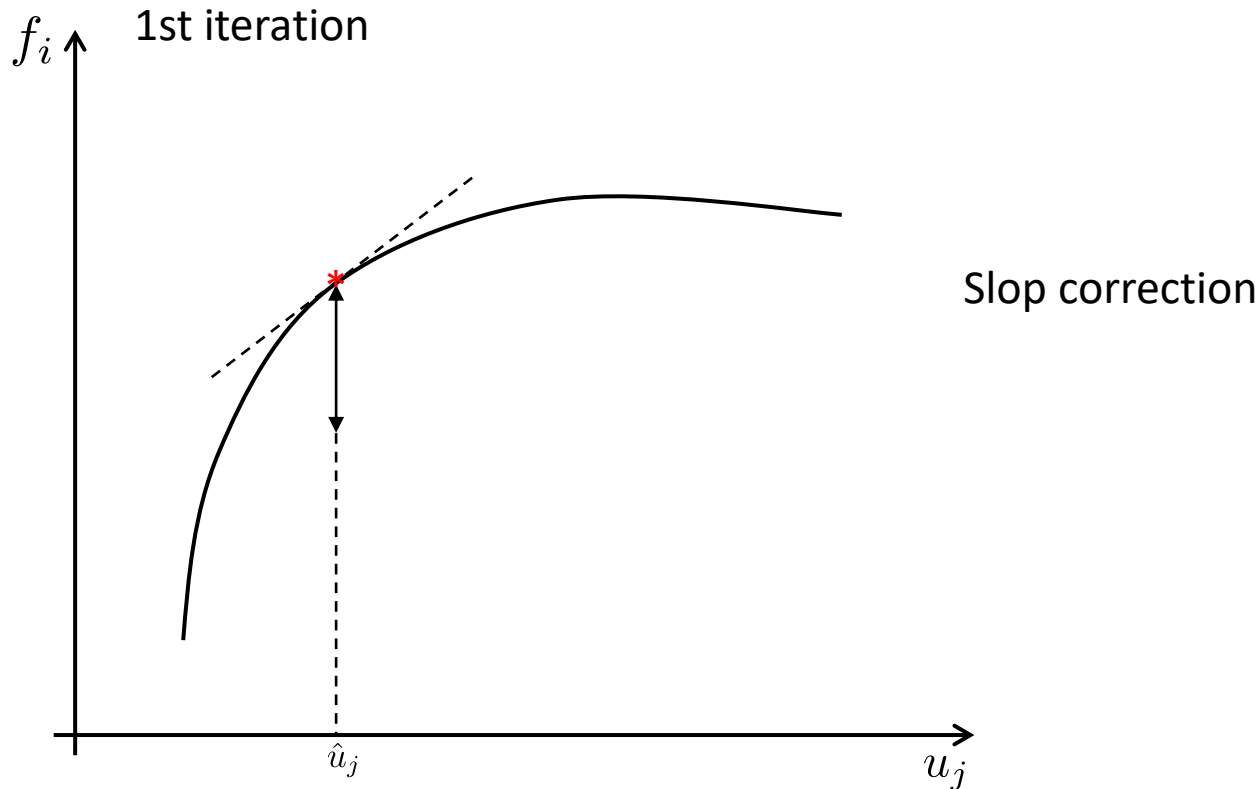
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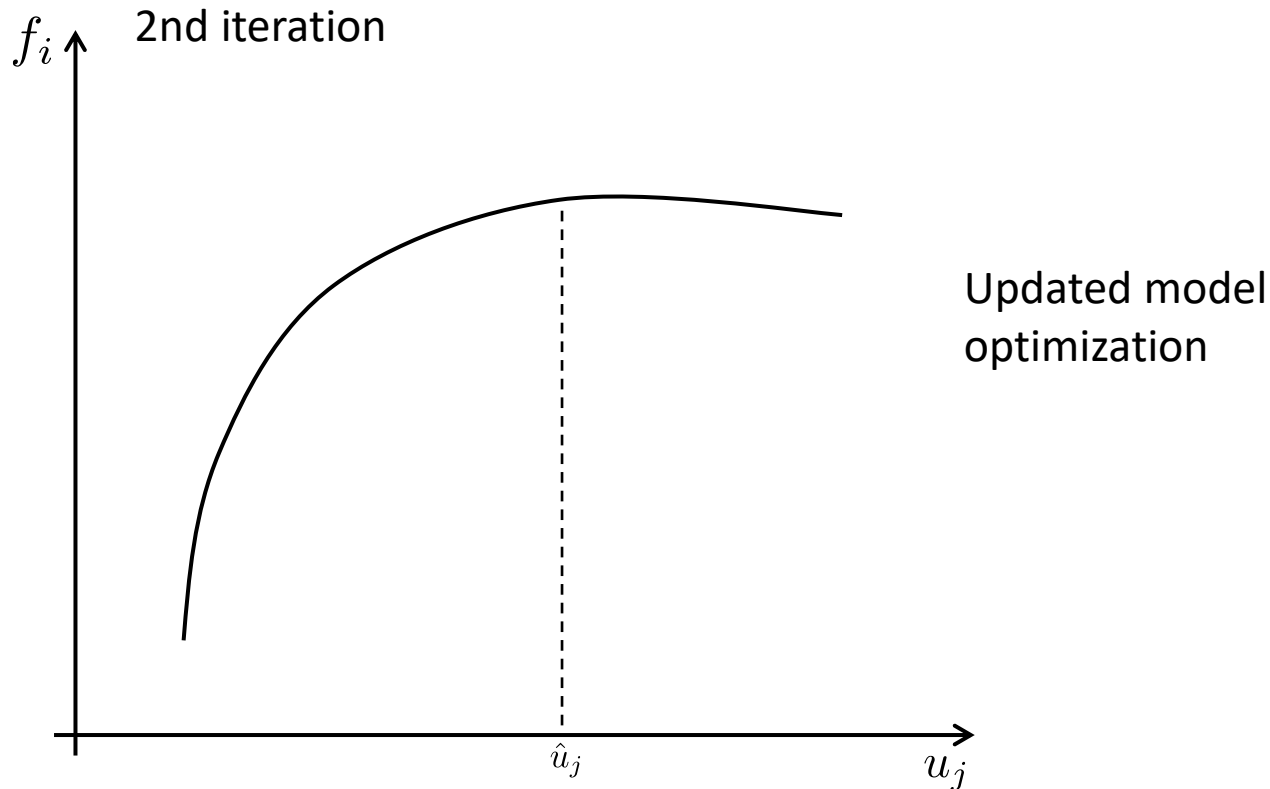




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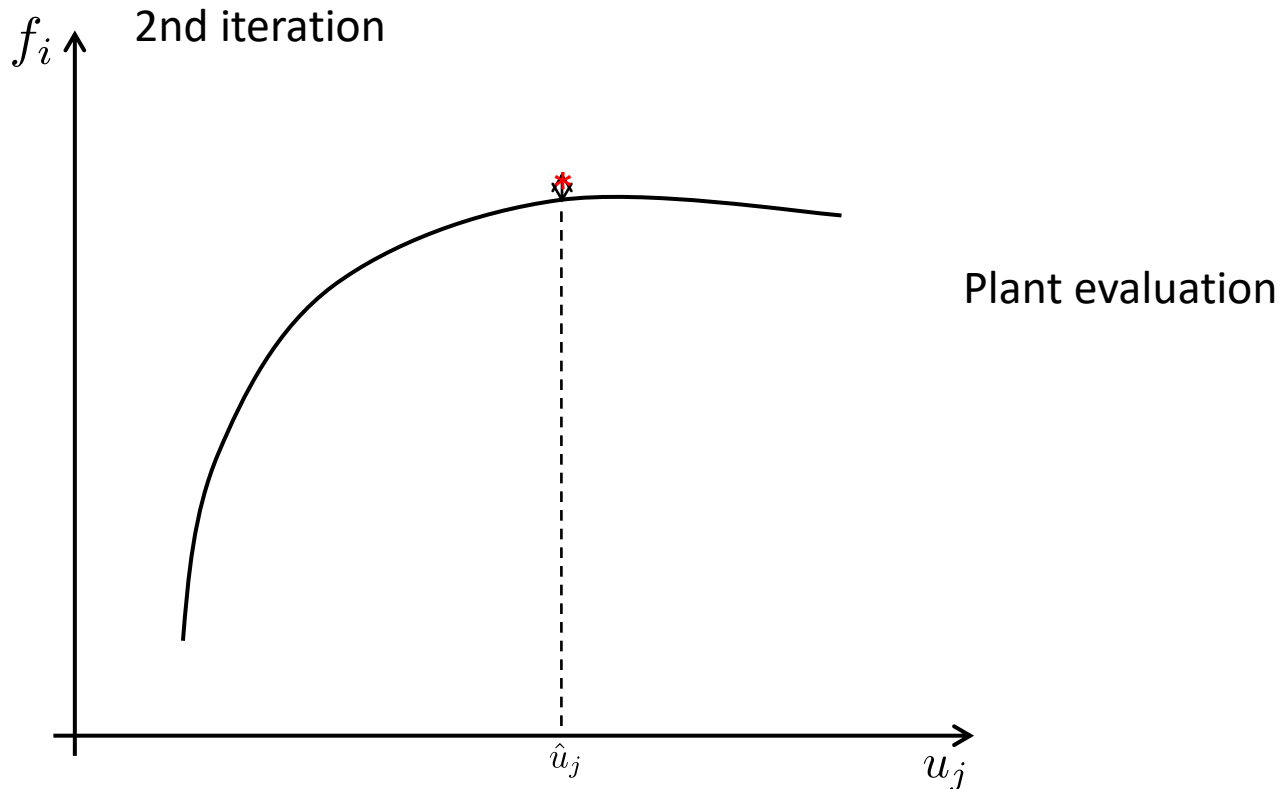
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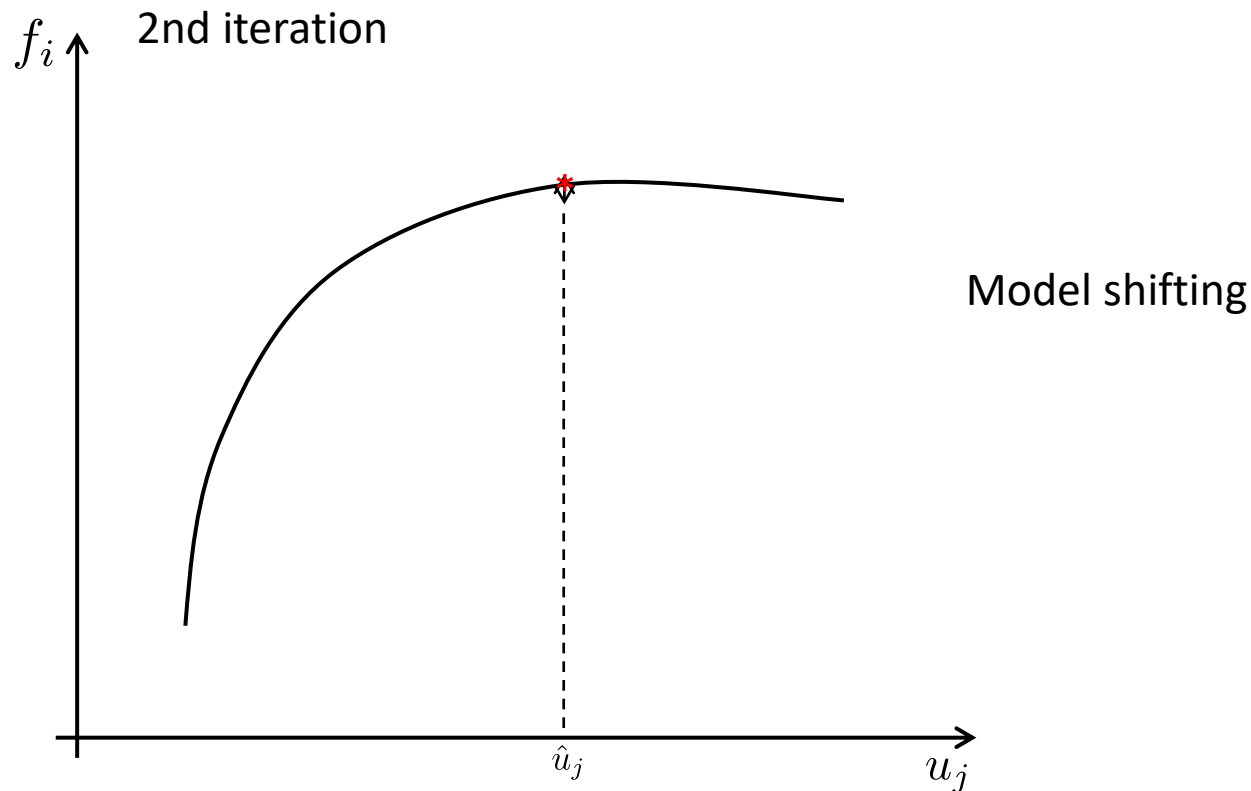
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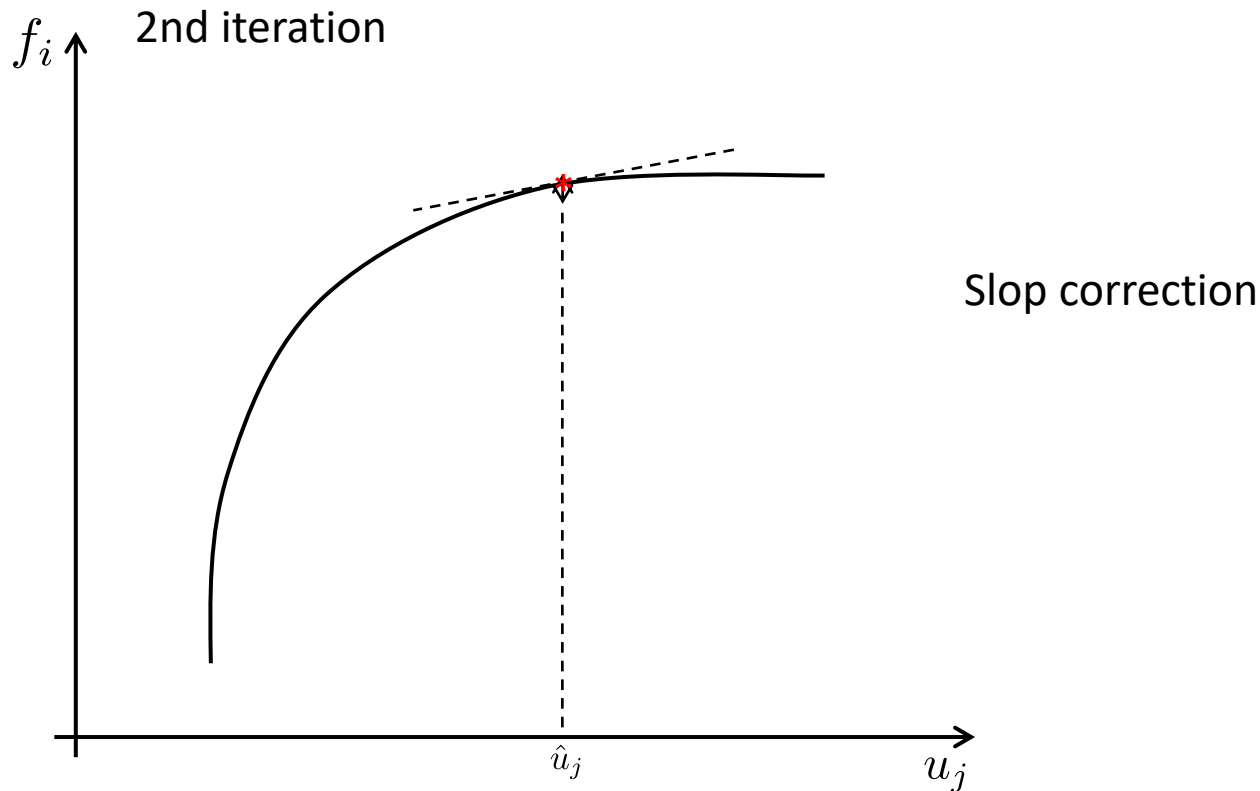
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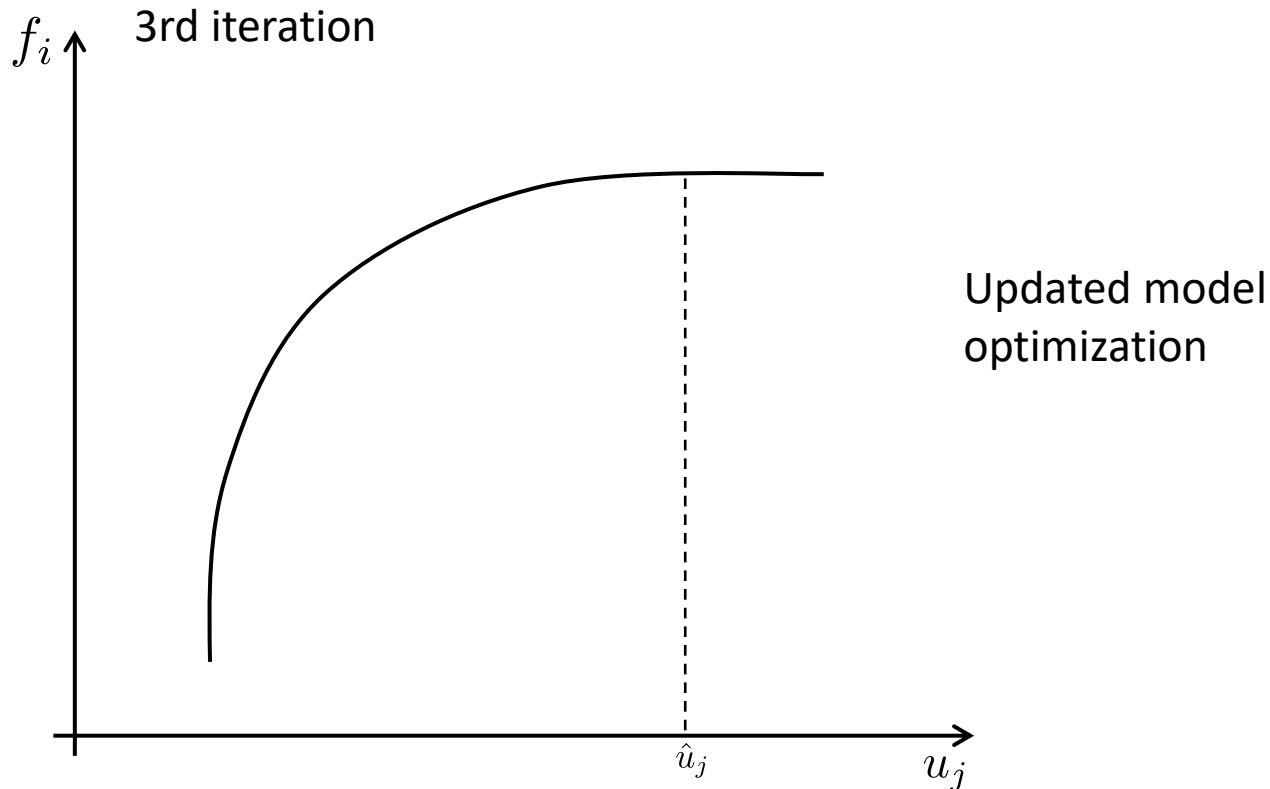
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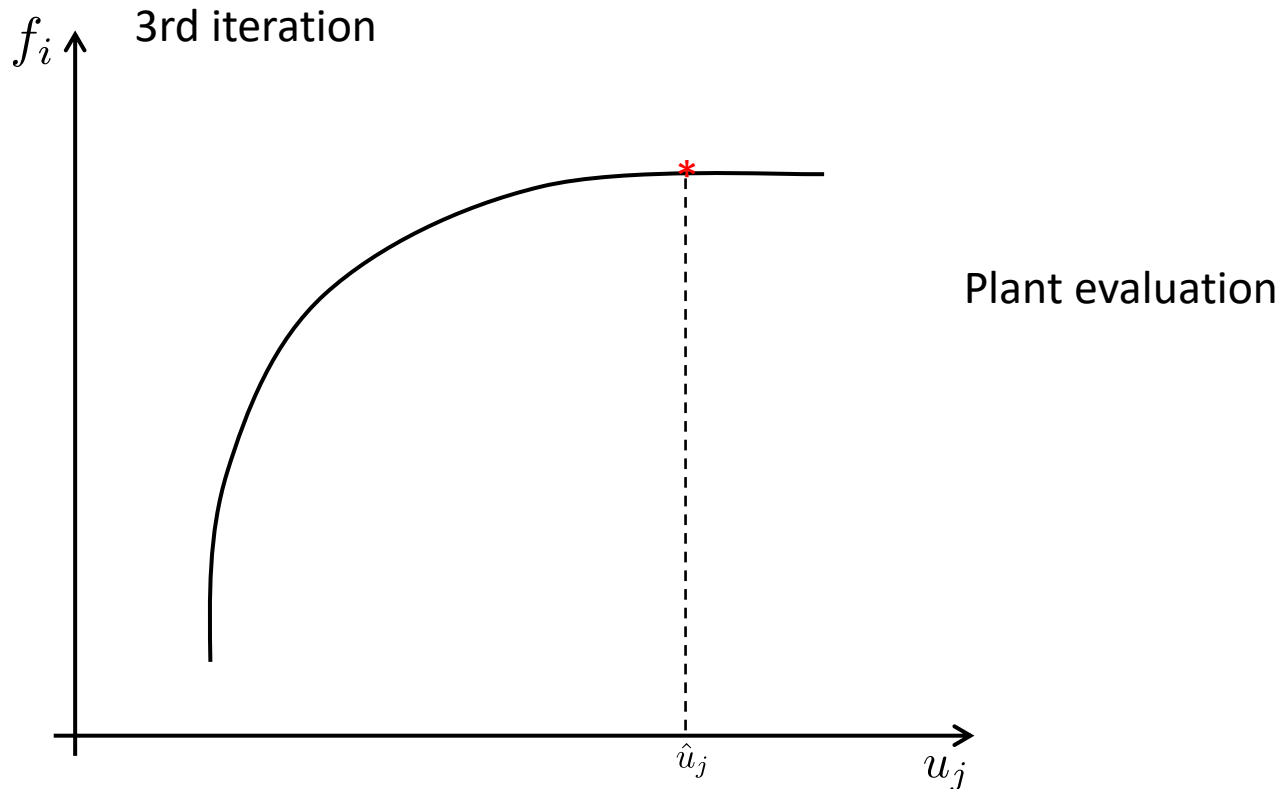
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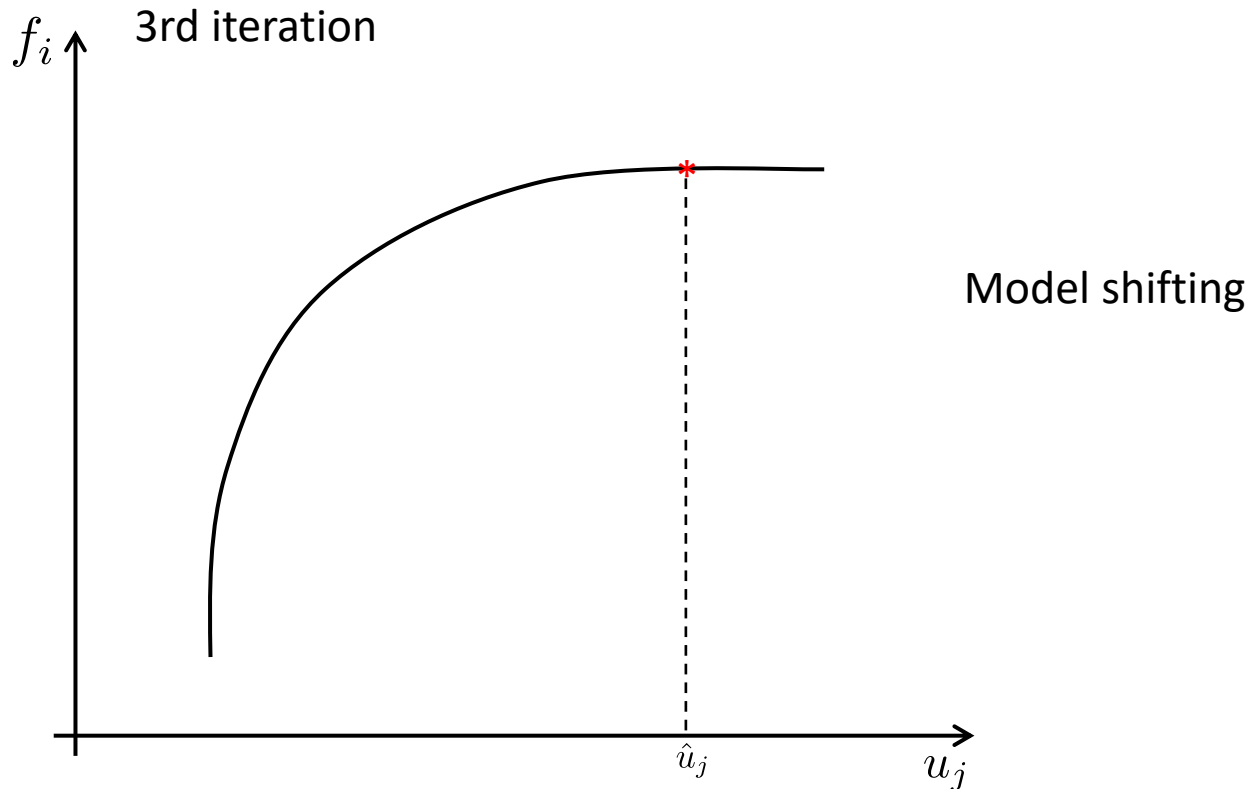
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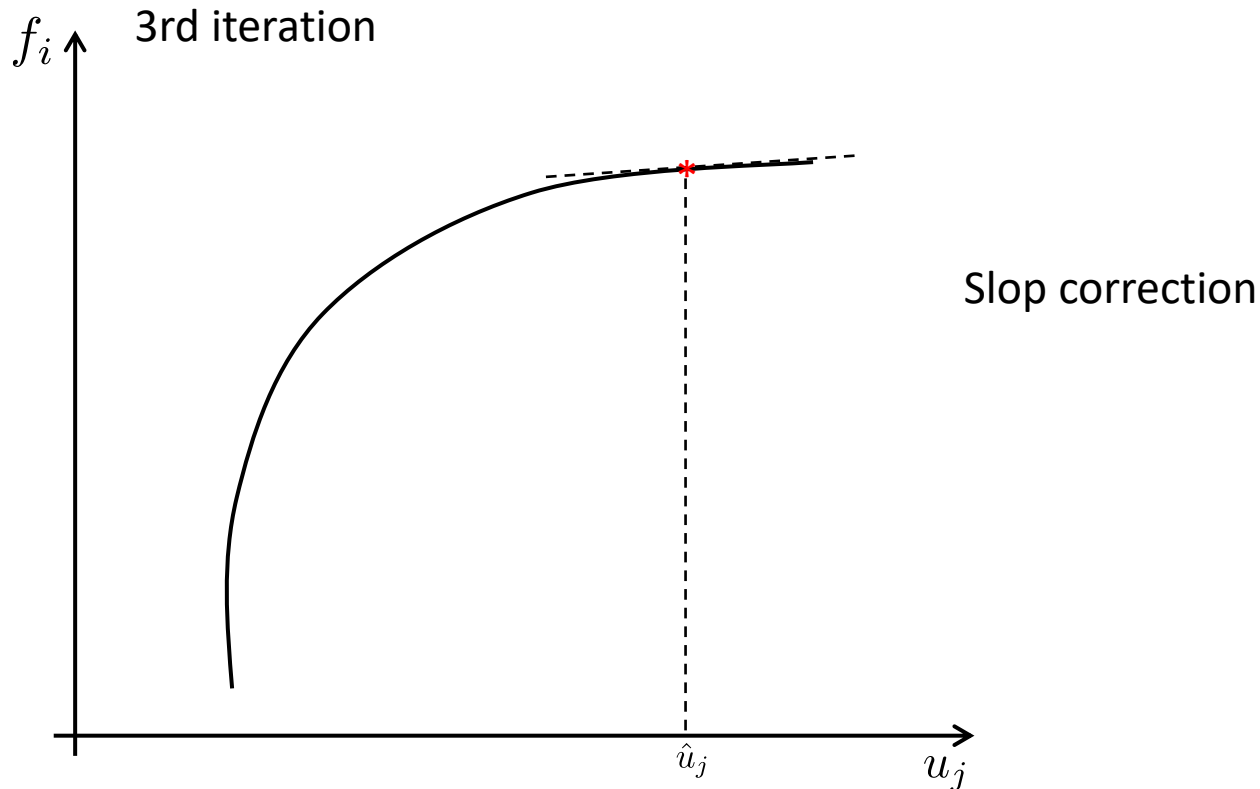
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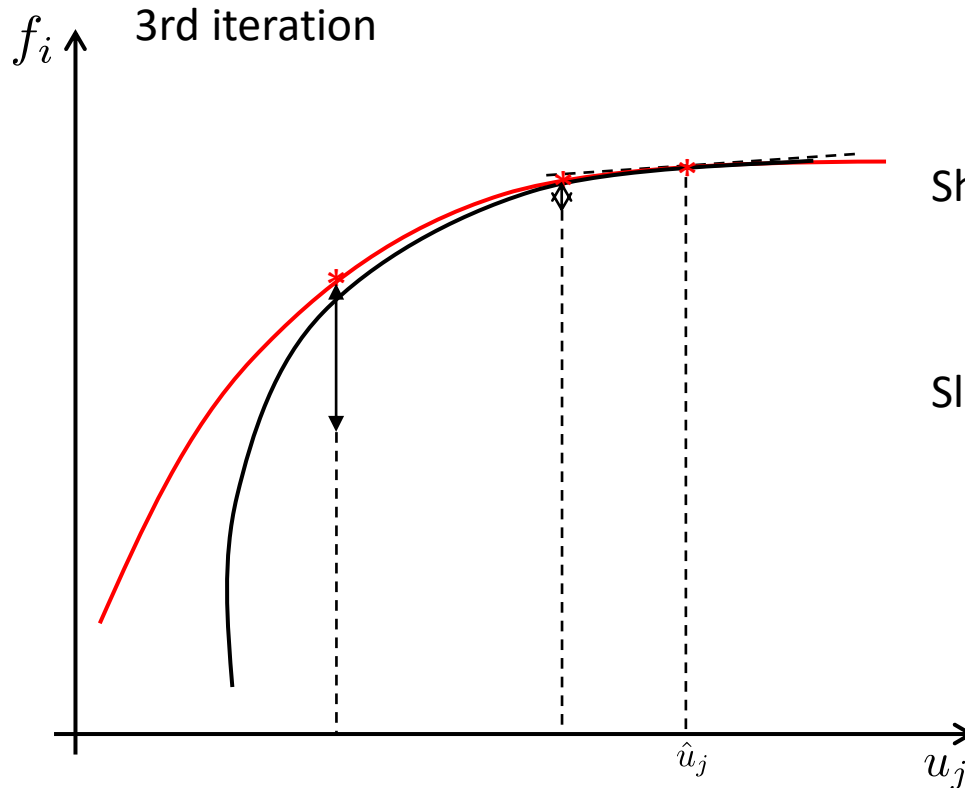




# Basic Features of Modifier Adaptation

How is the optimization scheme adapted?

Modifier Adaptation: Zeroth- and first-order correction terms added to cost and constraint functions to the optimization problem



Shifting correction:

$$\varepsilon_k^f = f_p(u_k) - f(u_k)$$

Slope correction:

$$\underbrace{\left( \frac{\partial f_p}{\partial u}(u_k) - \frac{\partial f}{\partial u}(u_k) \right)}_{\lambda^f} (u - u_k)$$

# Modifier-Adaptation Scheme

The modified optimization problem can be written as follows:

$$\begin{aligned} \mathbf{u}_{k+1}^* &\in \arg \min_{\mathbf{u}} \Phi_{m,k}(\mathbf{u}) \\ \text{s.t. } &G_{m,i,k}(\mathbf{u}) \leq 0 \\ &\mathbf{u}^L \leq \mathbf{u} \leq \mathbf{u}^U \end{aligned} \quad (1a)$$

Where the modified constraint and cost functions can be written as

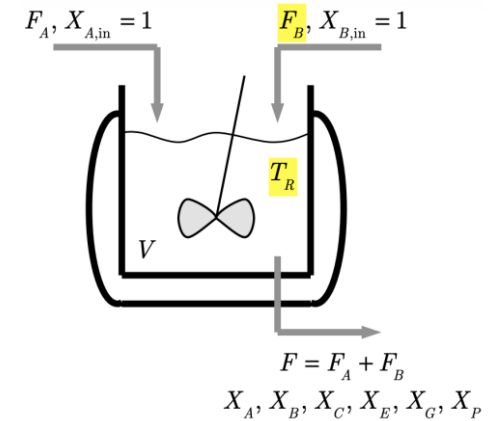
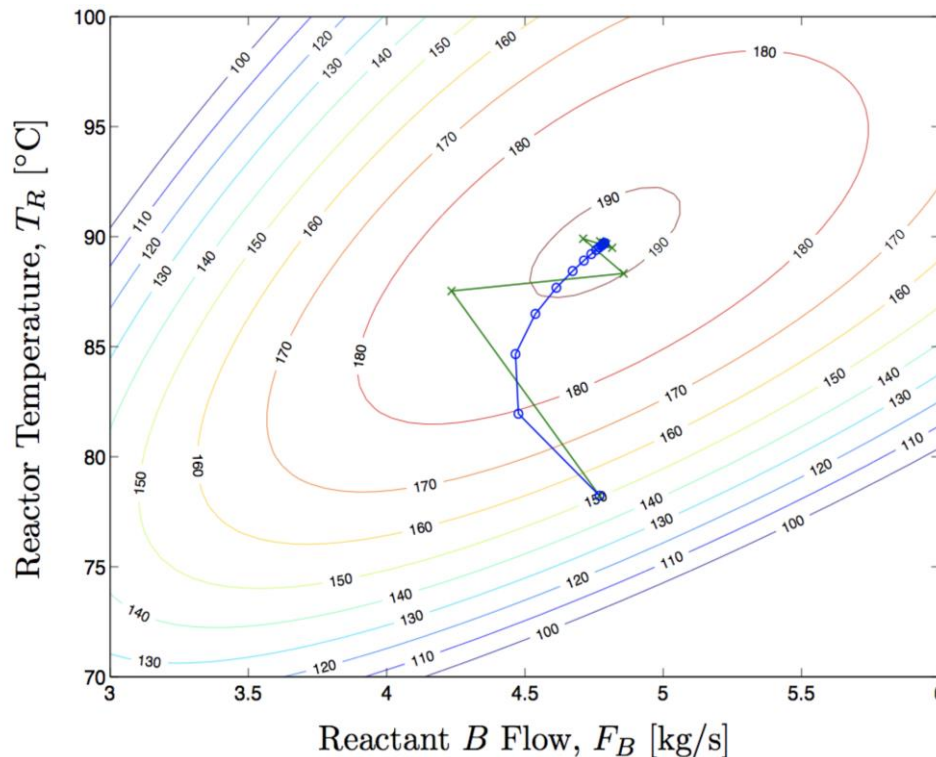
$$\begin{aligned} \Phi_{m,k}(\mathbf{u}) &:= \Phi(\mathbf{u}) + \varepsilon_k^\Phi + (\boldsymbol{\lambda}_k^\Phi)^\top (\mathbf{u} - \mathbf{u}_k) \\ G_{m,i,k}(\mathbf{u}) &:= G_i(\mathbf{u}) + \varepsilon_k^{G_i} + (\boldsymbol{\lambda}_k^{G_i})^\top (\mathbf{u} - \mathbf{u}_k) \leq 0, \quad i = 1, \dots, n_g, \end{aligned} \quad (1b)$$

The zeroth- and first-order modifiers are

$$\begin{aligned} \varepsilon_k^\Phi &= \Phi_p(\mathbf{u}_k) - \Phi(\mathbf{u}_k), \\ \varepsilon_k^{G_i} &= G_{p,i}(\mathbf{u}_k) - G_i(\mathbf{u}_k), \quad i = 1, \dots, n_g, \\ (\boldsymbol{\lambda}_k^\Phi)^\top &= \frac{\partial \Phi_p}{\partial \mathbf{u}}(\mathbf{u}_k) - \frac{\partial \Phi}{\partial \mathbf{u}}(\mathbf{u}_k), \\ (\boldsymbol{\lambda}_k^{G_i})^\top &= \frac{\partial G_{p,i}}{\partial \mathbf{u}}(\mathbf{u}_k) - \frac{\partial G_i}{\partial \mathbf{u}}(\mathbf{u}_k), \quad i = 1, \dots, n_g. \end{aligned} \quad (1c)$$

# KKT Matching

**Theorem 1** (MA convergence) KKT matching. *Consider the problem of optimizing a plant with an inaccurate yet adequate model using MA, let  $\mathbf{u}_\infty = \lim_{k \rightarrow \infty} \mathbf{u}_k$  be a fixed point of the MA iterative scheme. Then, not only  $\mathbf{u}_\infty$  is a KKT point of the modified model-based optimization Problem (1),  $\mathbf{u}_k$  is also a KKT point of the plant problem.*



Williams-Otto Reactor

- 4th-order model
- 2 inputs
- 2 adjustable param.

**Converges to  
plant optimum!**

# Model Adequacy Conditions

**Definition 1** (Model-adequacy criterion). *A process model is said to be adequate for use in an RTO scheme if it is capable of producing a fixed point that is a local minimum for the RTO problem at the plant optimum  $\mathbf{u}_p^*$ .*

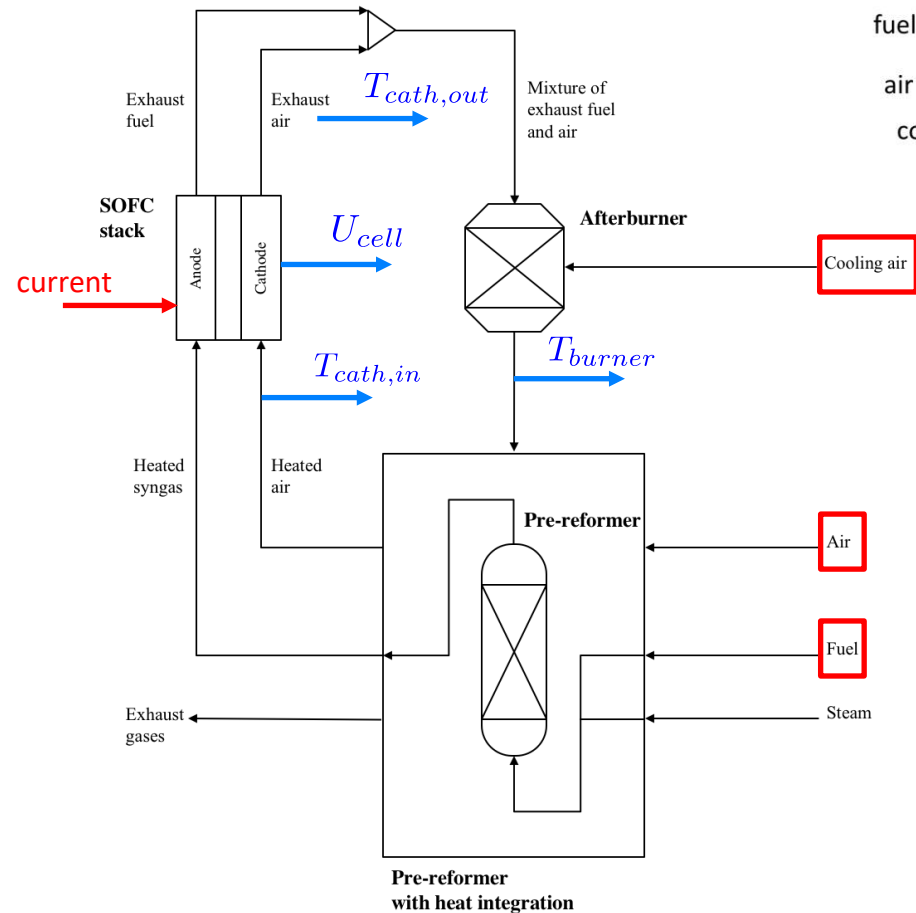
**Proposition 1** (Model-adequacy conditions for MA). *Let  $\mathbf{u}_p^*$  be a regular point for the constraints and the unique plant optimum. Let  $\nabla_r^2 \mathcal{L}(\mathbf{u}_p^*)$  denote the reduced Hessian of the Lagrangian of Problem (1) at  $\mathbf{u}_p^*$ . Then, the following statements hold:*

- i. if  $\nabla_r^2 \mathcal{L}(\mathbf{u}_p^*)$  is positive definite, then the process model is adequate for use in the MA scheme.*
- ii. If  $\nabla_r^2 \mathcal{L}(\mathbf{u}_p^*)$  is not positive semi-definite, then the process model is inadequate for use in the MA scheme.*
- iii. If  $\nabla_r^2 \mathcal{L}(\mathbf{u}_p^*)$  is positive semi-definite and singular, then the second-order conditions are not conclusive with respect to model adequacy.*

# Experimental Real-Time Optimization

# RTO of a Commercial SOFC System

## BlueGEN System



## Operational Constraints

Constraint	Lower bound	Upper bound
$U_{cell}$ [V]	0.76	-
$v$ [-]	-	0.8
$\lambda_{air}$ [-]	3	-
$T_{cath,in}$ [°C]	650	750
$T_{cath,out}$ [°C]	650	790
$T_{burner}$ [°C]	-	890
$q_{CH_4}$ [L.min <sup>-1</sup> ]	1	7
$q_{air}$ [L.min <sup>-1</sup> ]	85	200
$I$ [A]	0	50
$q_{cool}$ [L.min <sup>-1</sup> ]	0	40

# Optimization problem for the SOFC System

$$\max_{\mathbf{u}} \eta_{sys}(\mathbf{u}) = \frac{P_{el}}{q_{CH_4} LHV_{CH_4}}$$

s.t. steady-state model equations

$$P_{el}(\mathbf{u}) + \varepsilon^{P_{el}} = P_{el}^S [W]$$

$$U_{cell}(\mathbf{u}) + \varepsilon^{U_{cell}} \geq 0.76 [V]$$

$$650 \leq T_{cath,in}(\mathbf{u}) + \varepsilon^{T_{cath,in}} \leq 750 [^{\circ}C]$$

$$650 \leq T_{cath,out}(\mathbf{u}) + \varepsilon^{T_{cath,out}} \leq 790 [^{\circ}C]$$

$$T_{burner}(\mathbf{u}) + \varepsilon^{T_{burner}} \leq 890 [^{\circ}C]$$

$$\nu(\mathbf{u}) \leq 0.8$$

$$\lambda_{air}(\mathbf{u}) \geq 3$$

$$1 \leq q_{CH_4} \leq 7 [L.min^{-1}]$$

$$85 \leq q_{air} \leq 200 [L.min^{-1}]$$

$$0 \leq I \leq 50 [L.min^{-1}]$$

$$0 \leq q_{cool} \leq 40 [L.min^{-1}]$$

## Electrical Power Profile 1:

$$\tau_{RTO} = 90 \text{ min}$$

$$P_{el}^S(t) = \begin{cases} 1000 [W], & t \leq 24 \text{ h} \\ 1250 [W], & 24 \text{ h} < t \leq 48 \text{ h} \\ 1500 [W], & t > 48 \text{ h}. \end{cases}$$

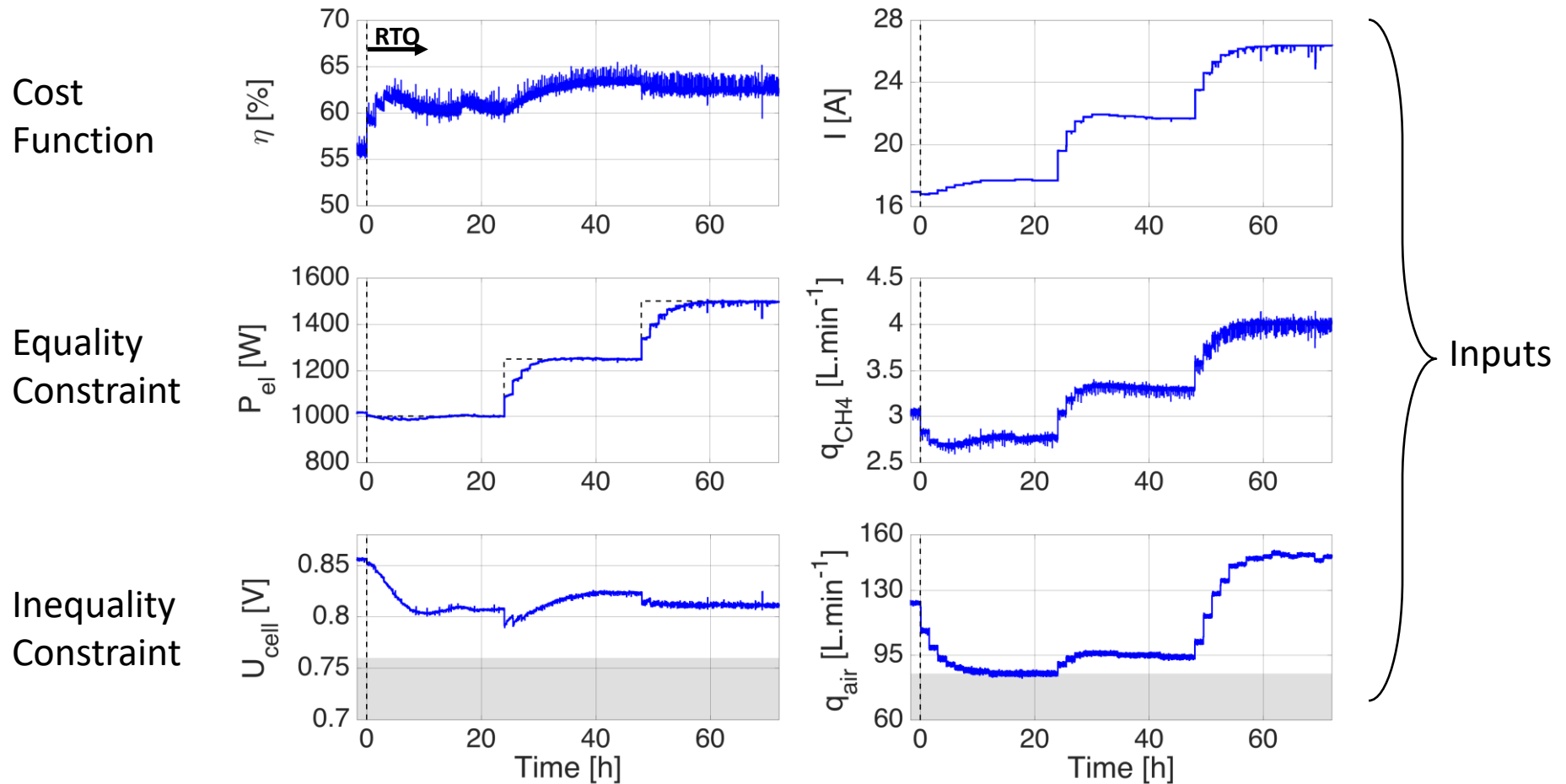
## Electrical Power Profile 2:

$$\tau_{RTO} = 5 \text{ min}$$

$$P_{el}^S(t) = \begin{cases} 1000 [W], & t \leq 2 \text{ h} \\ 1250 [W], & 2 \text{ h} < t \leq 4 \text{ h} \\ 1500 [W], & t > 4 \text{ h}. \end{cases}$$

# Experimental Results: Steady-State CA (Profile 1)

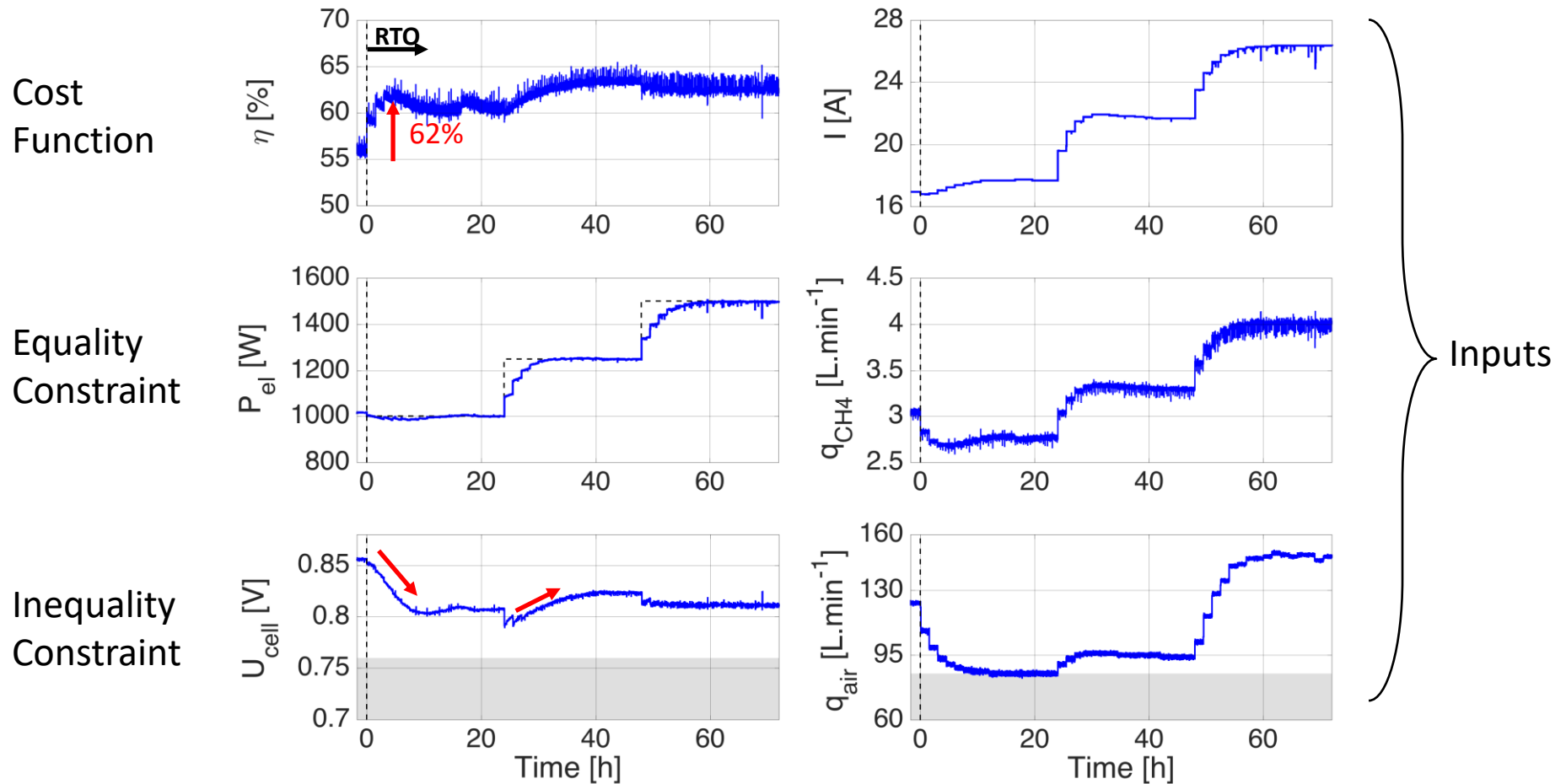
## Exp. 1: $CA_{ss}$ , $\tau_{RTO} = 90$ min and 72-h Experiment





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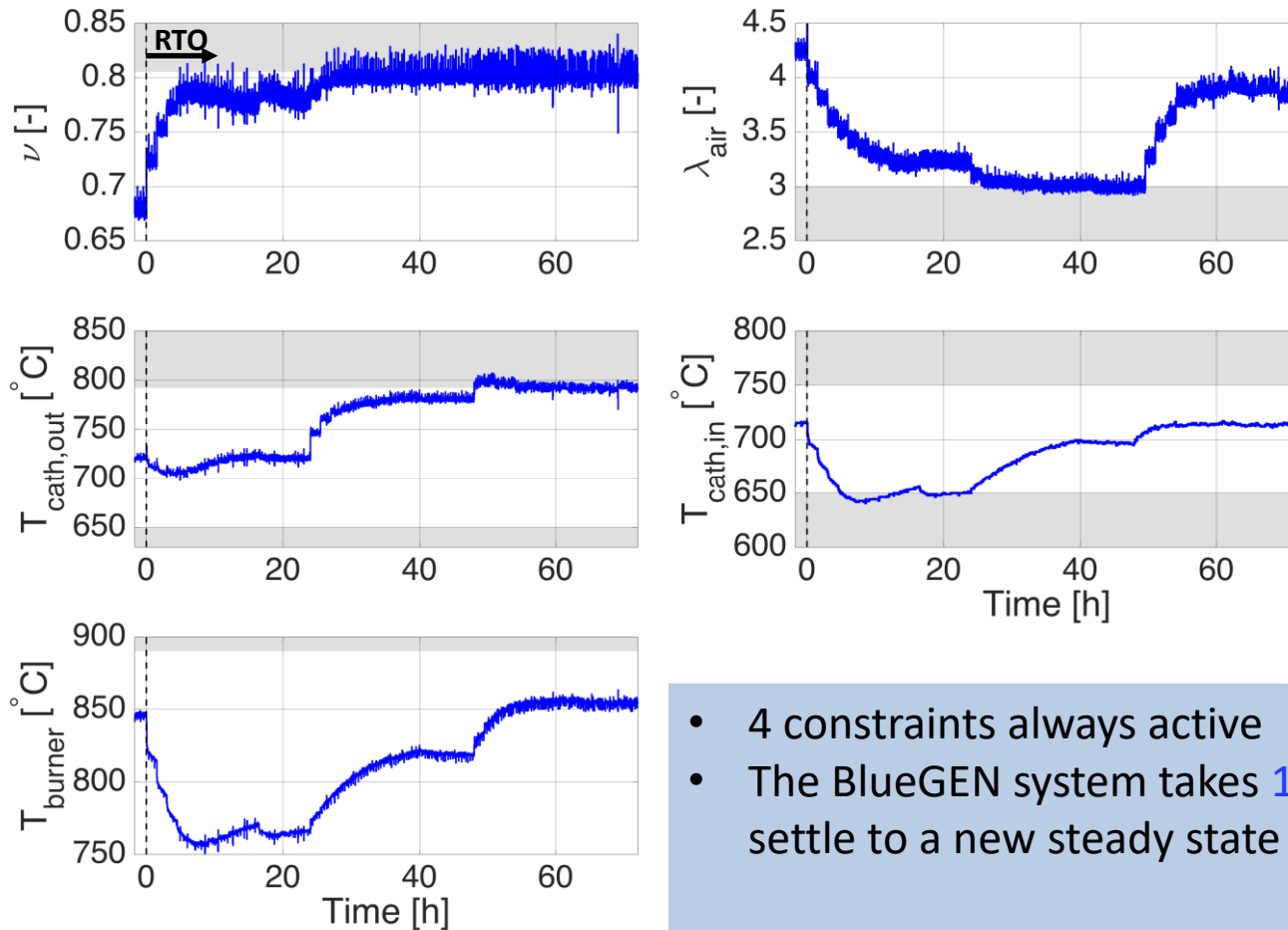
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## Inequality Constraints

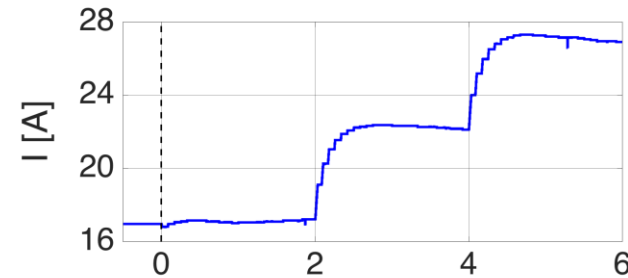
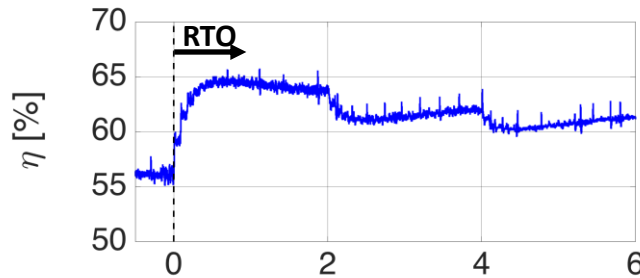


- 4 constraints always active
- The BlueGEN system takes 16 to 20 h to settle to a new steady state

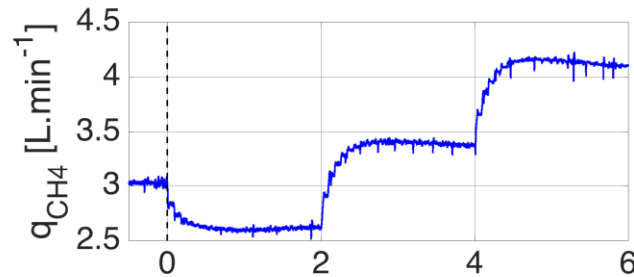
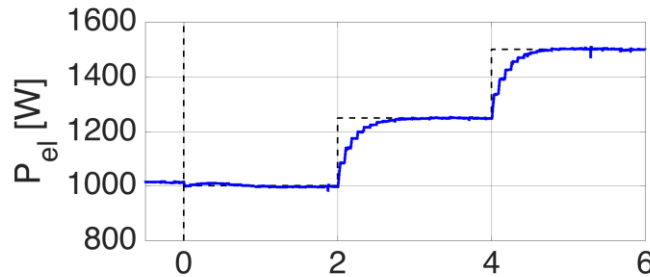
# Experimental Results: Fast CA (Profile 2)

## Exp. 2: CA\_dyn, $\tau_{RTO} = 5$ min and 6-h Experiment

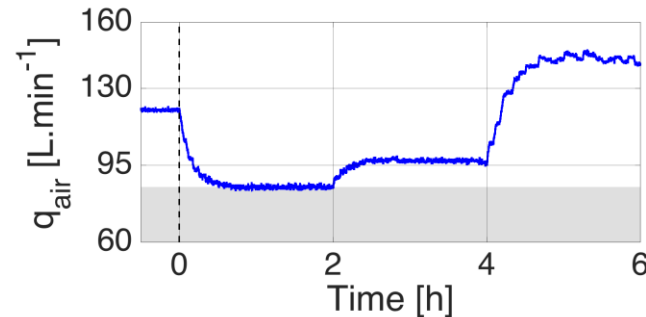
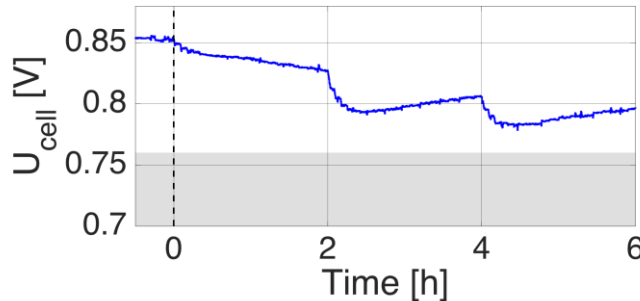
Cost  
Function



Equality  
Constraint



Inequality  
Constraint

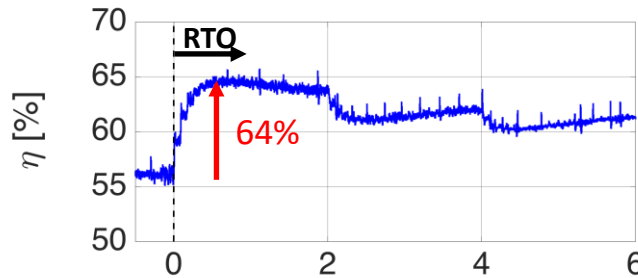


Inputs

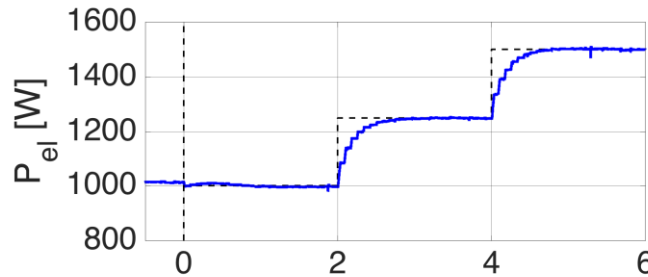
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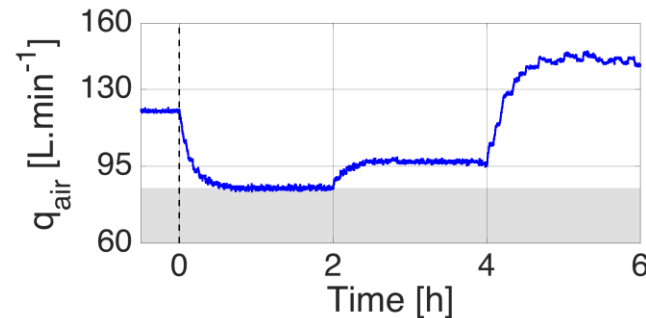
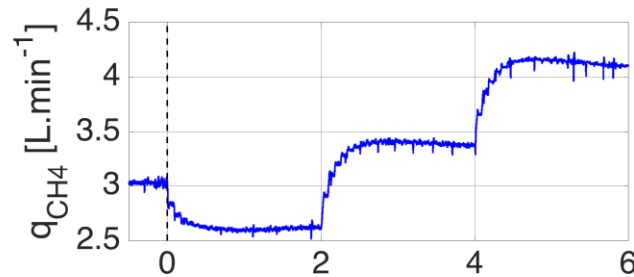
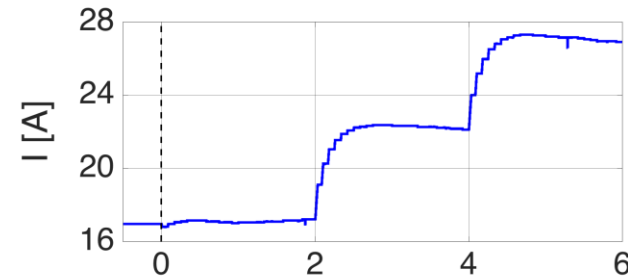
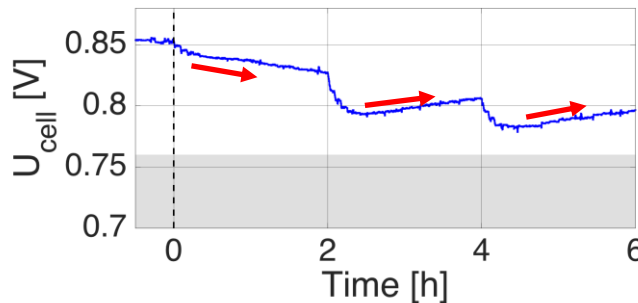
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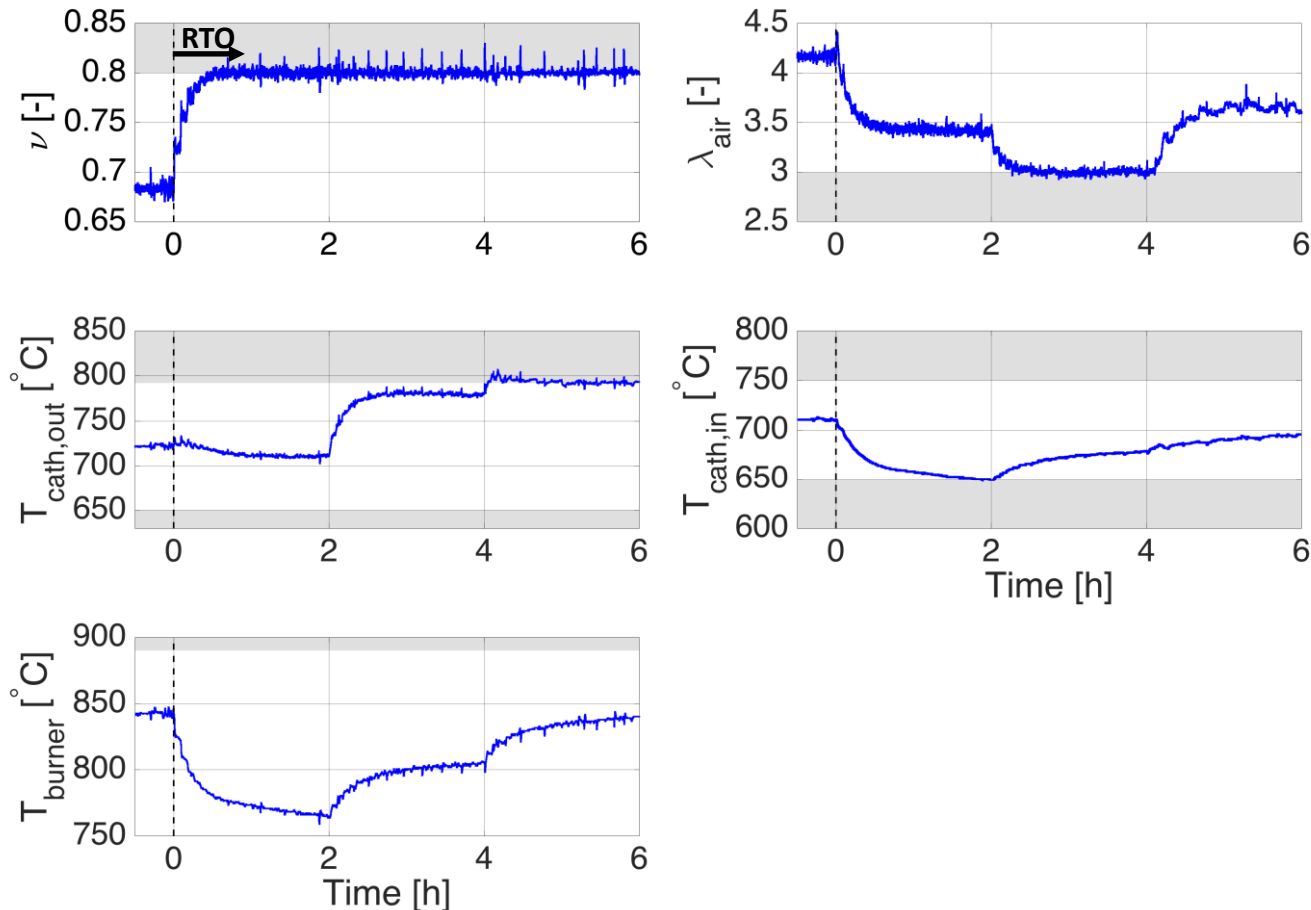


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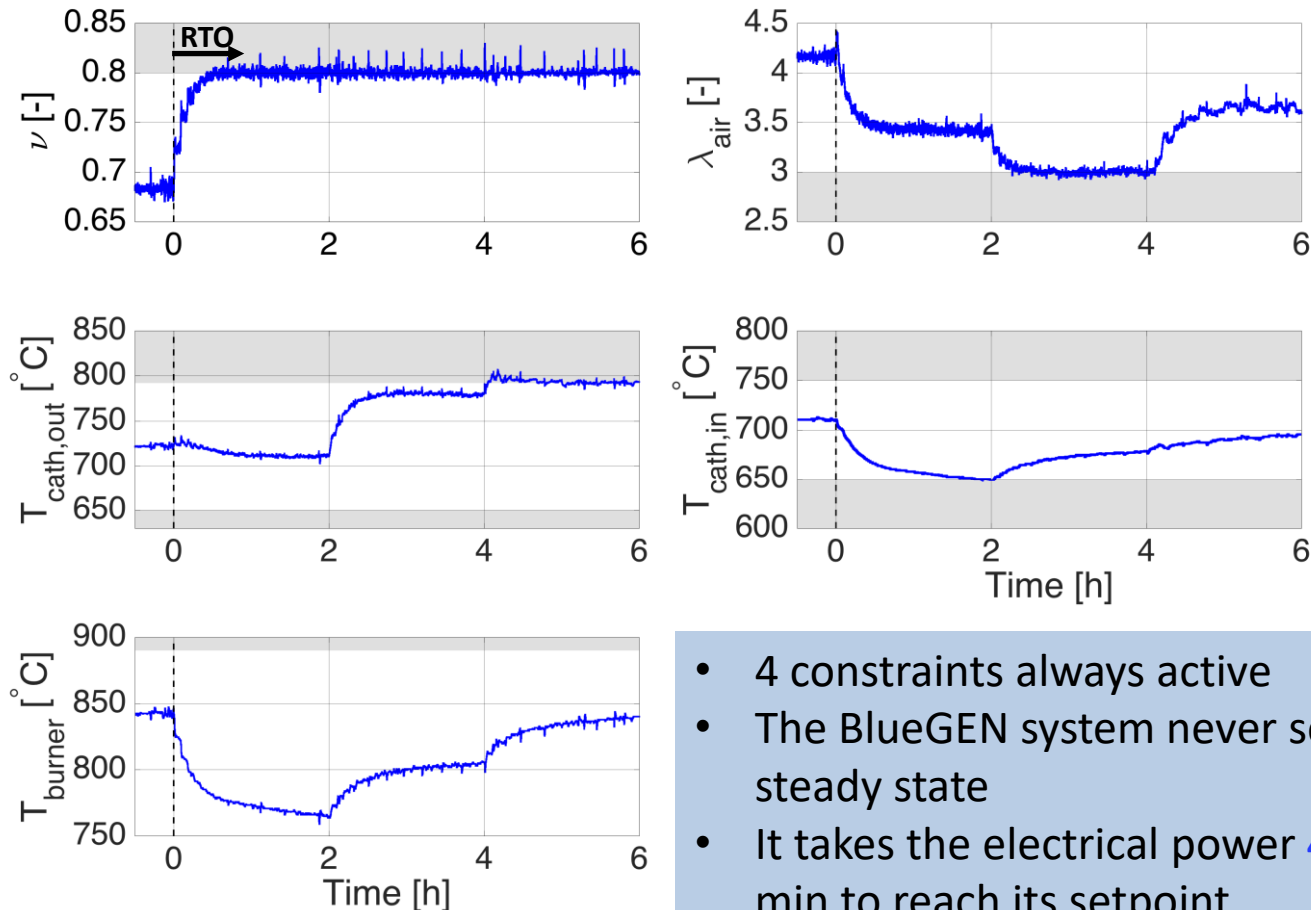
### Inequality Constraints



# Experimental Results: Fast CA (Profile 2)

## Exp. 2: CA\_dyn, $\tau_{RTO} = 5$ min and 6-h Experiment

### Inequality Constraints



- 4 constraints always active
- The BlueGEN system never settles to a steady state
- It takes the electrical power 40 to 45 min to reach its setpoint

# Conclusions

- RTO is a family of optimization methods that incorporate process measurements in the optimization framework to drive a real process to optimal performance
- We develop RTO approaches that tackle specific targets defined by industry requirements as well as proving their properties and experimental application for validation
- RTO is suited for a broad range of industrial processes, including fuel cells and degrading systems
- The main features of RTO includes the ability of reaching plant optimality and constraint satisfaction
- A variant of modifier-adaptation has been developed and applied to a commercial system (SOLIDpower)
- We will apply RTO in both RUBY (SOFC) and REACTT (SOE)

# Thank you!



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